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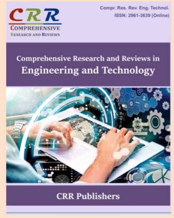

(RESEARCH ARTICLE)

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# Three-level laser quantum optics system coupled to thermal reservoirs

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## Abstract

In this study, we investigate the quantum characteristics of a three-level system coupled to a two-mode thermal reservoir. We employ stochastic differential equations, which incorporate normal ordering, to describe the dynamics of the system. Our primary focus is on examining the squeezing and entanglement properties of the two-mode cavity light. The results of the study demonstrate a significant level of entanglement between the two cavity modes. Furthermore, there exists a direct correlation between the degree of entanglement and the degree of two-mode squeezing. This discovery emphasizes the potential for enhancing the non-classical properties of the system by utilizing nonlinear optical elements. The study provides valuable insights into the interplay between squeezing, entanglement, and thermal noise in a three-level system coupled to a two-mode thermal reservoir. The findings suggest that the introduction of nonlinear elements can enhance the squeezing and entanglement of the system, even in the presence of thermal noise.

**Keyword:** Laser; Photon; Quadrature variation; Superposition; Thermal reservoir

## 1 Introduction

A three-level laser quantum optics system, coupled to thermal reservoirs, refers to a setup in which a three-level quantum system, such as a three-level atom or a quantum dot, is utilized as the active medium in a laser. The system is connected to thermal reservoirs, which represent the surrounding environment and can induce thermal fluctuations in the system. In this configuration, the three energy levels of the quantum system are typically labeled as the ground state, the excited state, and an intermediate state. The system can be stimulated from the ground state to the excited state using an external energy source, such as an optical or electrical pump. Subsequently, the excited state decays to the intermediate state through spontaneous emission, and the system can return to the ground state through various relaxation processes.

The thermal reservoirs represent the thermal fluctuations in the environment, which can cause the system to undergo transitions between its energy levels. These transitions can lead to the emission of thermal photons, contributing to the overall noise in the system. The interaction between the quantum system and the thermal reservoirs can be described using quantum optics formalism, such as the master equation or the quantum Langevin equation. The coupling to thermal reservoirs introduces various effects in the system, including relaxation processes, phasing, and thermal noise. These effects can impact the coherence and dynamics of the system, affecting properties such as the emission spectrum, line width, and photon statistics of the laser output.

Understanding and controlling the interaction between the three-level quantum system and the thermal reservoirs is crucial for optimizing the performance of the laser. It also has implications for applications such as quantum information processing, where noise and coherence can degrade the performance of quantum operations. Overall, the investigation of a three-level laser quantum optics system coupled to thermal reservoirs involves studying the interplay between the quantum dynamics of the system and the thermal fluctuations in the environment. This research area is significant for

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both gaining a fundamental understanding of quantum systems and applying it to laser technology and quantum information science. The study of the density and structural computations of light generated by three-level lasers has received significant attention [1–17]. Three-level atoms are initially arranged in a parallel surface of two distinct levels and then injected into a hole connected to a vacuum dam by a single-hole mirror. This process can be seen as a quantum vision system for a three-level laser. The system with three distinct levels consists of two dipole transients. There are three different types of three-level systems, named Lambda Λ, Cascade, and Vee (V), each combining different exciting circumstances and mid-level states. Three-level atoms are steadily fed into a hole in a three-level laser and released after a predetermined amount of time (τ T). The three levels are represented by the symbols |a>, |b>, and |c>.

The study of the interaction between a laser system and its surrounding thermal environment involves three-level lasers connected to a thermal reservoir. The temperature environment is described by a statistical ensemble, and the laser system is addressed in a quantum mechanical fashion. The first step in analyzing this system is typically defining the Hamiltonian that controls the laser system's dynamics. Components in this Hamiltonian represent the laser's energy levels, the coupling between these levels, and the interaction with the thermal environment. To describe the laser system, a series of density matrix equations that follow the evolution of the populations and coherences of the energy levels are typically used.

To simulate the thermal environment, a density matrix representing the statistical distribution of its energy states is used, taking into consideration the temperature and other pertinent environmental factors. A master equation is commonly used to explain the temporal evolution of the laser system's density matrix and the interaction between the laser system and the thermal environment. By solving this master equation, numerous features of the three-level laser system linked to the thermal quantum state may be studied, including the lasers' steady-state. performance, including their output power and efficiency. It also sheds light on the ways in which thermal fluctuations influence the behavior of the laser system. Knowledge and improving the performance of lasers in real-world applications—where thermal effects can have a substantial impact on their operation—requires knowledge of this methodology [1, 3-13] concentrated light has the potential to work in low-noise communication and weak signal detection. A three-level laser can be described as a quantum optical system in which atoms with three levels, are first arranged at a high relative level of two levels, injected into a hole. When a three-level atom makes a transition from top to bottom, two ports are produced. The combination of the two levels of the atom is responsible for the exciting non-artificial features of the light produced. Typically, an atomic fusion can be made of a three-level atom by attaching a corresponding light or by arranging an atom at the beginning of the upper junction of these two levels. There are a number of quantum optical systems that can produce light with non-artificial features such as compression, folding, and anti-bonding etc.

We assume that the direct fluctuations between levels | a> → | c> reject the hole modes corresponding to two fluctuations | a> ↔ | b> and | a> ↔ | b>. This is believed to be the initial state in which atoms are repaired at the beginning of | a> and | c>. When a three-level atom decomposes in a cascade from top to bottom, it releases two photons. If these two photons have identical frequencies, the three-level atom is called a degenerate three-level atom; otherwise, it is referred to as a non-degenerate three-level atom. It is understood that the two photons generated by the cascade three-level laser system exhibit squeezing properties under certain conditions due to the correlation between the photons.

## 2 Materials and methods

### 2.1 Three-Level Atom with Radiation

In this chapter, we aim to develop a Hamiltonian description of radiation interactions in the Vee three-level atom. We will combine this with the master three-level laser equation and a vacuum reservoir. [8]

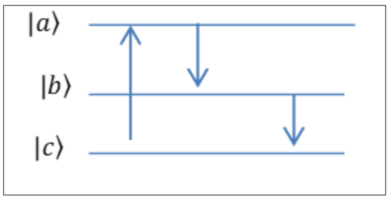


**Figure 1** Schematic diagram of cascade -type three-level laser

## 2.2 The Hamiltonian

In this section we seek to drive the Hamiltonian describing the interaction of a radiation with three-level atom. The interaction of a one-electron atom having mass m and charge e with a single-mode radiation represented by the vector potential A, is described by the Hamiltonian [1-3]

$$\hat{H}_1 = \frac{e}{m}\hat{p}\,.\hat{A}\;\; \frac{e^2}{2m}\hat{A}^2 \qquad (1)$$

Where $\hat{P}$ is the canonical momentum? Since, we can neglect the second term in

Eqn.(2.1). Hence the interaction Hamiltonian can be put in the form

$$\hat{H}_1 = \frac{e}{m}\hat{p}\,.\hat{A} \qquad (2)$$

It proves to be more convenient to express this Hamiltonian in terms of electric field operator. To this end, we note that

$$\frac{d}{dt}\left(\hat{r}.\hat{A}\right) = -\frac{ie}{m\hbar}\left[\hat{r}.\hat{A}\,,\hat{p}\,.\hat{A}\right] \qquad (3)$$

n which $\hat{r}$ is the position operator for the electron. Taking into account the fact that the vector potential commute with the position and momentum operators, one readily finds [11]

$$\left[\hat{r}.\hat{A}\,,\hat{p}\,.\hat{A}\right] = i\hbar\hat{A}^2\;. \qquad (4)$$

It then follows that

$$\frac{d}{dt}\left(\hat{r}.\hat{A}\right) = \quad \frac{e}{m}\hat{A}^2 \qquad (5)$$

On the other hand, we have

$$\frac{d}{dt}\left(\hat{r}.\hat{A}\right) = \hat{A}.\,\frac{d\hat{r}}{dt} + \hat{r}.\frac{d\hat{A}}{dt}\quad, \qquad (6)$$

so that combination of Eqn. (2.5) and (2.6) leads to

$$\hat{A}.\frac{d\hat{r}}{dt} = \frac{e}{m}\hat{A}^2 - \hat{r}.\frac{d\hat{A}}{dt} \qquad (7)$$

Furthermore, with the aid of the relation

$$\hat{p} = \mathrm{m}\frac{d\hat{r}}{dt} - \mathrm{e}\hat{A}, \qquad (8)$$

One can write

$$\hat{p}\cdot\hat{A} = \mathrm{m\ d}\frac{d\hat{r}}{dt}\cdot\hat{A} - \mathrm{e}\hat{A}^{\,2.} \qquad (9)$$

Now on account of Eqn. (2.7) and (2.9), we have

$$\hat{p}\cdot\hat{A} = -\,\mathrm{m}\hat{r}\cdot\frac{d\hat{A}}{dt}. \qquad (10)$$

In view of this the interaction the Hamiltonian described by Eqn. (2) can therefore be

Put in the form

$$\hat{H}_{\mathrm{I}} = -\mathrm{e}\hat{r}\cdot\frac{d\hat{A}}{dt}. \qquad (11)$$

It can be easily established that

$$\frac{d\hat{A}}{dt} = \frac{\partial\hat{A}}{\partial t} + (-\,\vec{v}\cdot\nabla)\hat{A} \qquad (12)$$

A simpler form of Eq. (12) can be obtained by applying the electric dipole approximation

$$e^{ik.r} = 1 \qquad (13)$$

This is evidently justified d provided that k.r << 1. Inspection of the vector potential in the electric dip ole approximation, on the position c o ordinates , it then turns out in this approximation that

$$(-\vec{v}\cdot\nabla)\hat{A} = 0\ , \qquad (14)$$

and hence in the Coulomb gauge we have

$$\frac{d\hat{A}}{dt} = \frac{\partial\hat{A}}{\partial t} = -\hat{E}. \qquad (15)$$

In view of this result, we see that

$$\hat{H}_I = e\hat{r}(t)\cdot\hat{E}\,(x,t). \qquad (16)$$

The electric field operator for two-mode light in the electric dipole approximation has the form

$$\hat{E}(x,t) = ie\left\{\left[\frac{\hbar\omega_a}{2\mathcal{E}_{oV}}\right]^{\frac{1}{2}}\left(\hat{a}\,e^{-i\omega_a t} - \hat{a}^{+}\,e^{i\omega_a t}\right) + \left\{\left[\frac{\hbar\omega_b}{2\mathcal{E}_{oV}}\right]^{\frac{1}{2}}\left(\hat{b}\,e^{-i\omega_b t} - \hat{b}^{+}\,e^{i\omega_b t}\right)\right\}\mathbf{u}. \qquad (17)$$

And

$$\hat{r}(t) = r_{ab}\left(\hat{\sigma}_{ab}\,e^{i\omega'_a t} + \hat{\sigma}_{ba}\,e^{-i\omega'_a t}\right) + r_{bc}\left(\hat{\sigma}_{bc}\,e^{i\omega'_b t} + \hat{\sigma}_{cb}\,e^{-i\omega'_b t}\right). \qquad (18)$$

The result is

$$\hat{H}_I = i\hbar g\begin{bmatrix}\left(\hat{\sigma}_{ab}\,\hat{a}e^{-i(\omega_a-\omega'_a)t} - \hat{\sigma}_{ba\ \hat{a}^{+}}e^{-i(\omega_a-\omega'_a)t}\right) + \\ \left(\hat{\sigma}_{bc\ \hat{b}}e^{i(\omega_b-\omega'_b)t} + \hat{\sigma}_{cb}\,\hat{b}^{+}e^{-i(\omega_b-\omega'_b)t}\right)\end{bmatrix}. \qquad (19)$$

## 3 Result and discussion

### 3.1 Master Equation

In this section we determine the master equation of a three-level lambda type in a two-mode vacuum storage facility.

#### *3.1.1 Cascade Three Laser Type*

We consider three-level lasers in which atoms with three levels in the V-shape are injected at a constant rate and released from the hole after a certain amount of time τ. We mean the upper and lower levels of the three-dimensional atom by | b>, | a>| c> as shown in Fig. 2

We assume that cavity pathways will be compatible with two versions | b> ↔ | a> and | b> ↔ | c>, there is a permissible dipole and a direct transition between levels | a> ↔ | c> for dipole rejection [11-13] .The atomic interaction of the three V-level and cavity pathways can be described in the Hamiltonian interaction image.

$$\hat{H}_I = ig\left[\left(\hat{\sigma}_{ab}\,\hat{a} - \hat{\sigma}_{ba\ \hat{a}^{+}}\right) + \left(\hat{\sigma}_{bc\,\hat{b}} + \hat{\sigma}_{cb}\,\hat{b}^{+}\right)\right] \qquad (20)$$

We take the initial state of a three-level atom to be

$$\psi A(0) = Ca(0)|\,a> + Cc(0)|c> \qquad (21)$$

The master equation for V three-level laser coupled to a two-mode vacuum reservoir can be Written as

$$\frac{d}{dt}\hat{\rho} = \frac{1}{2}\left(\kappa + B\rho_{aa}^{(0)}\right)\left(2\hat{a}\hat{\rho}\hat{a}^{+} - \hat{a}^{+}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{+}\hat{a}\right) + \frac{1}{2}\left(\kappa + B\rho_{cc}^{(0)}\right)\left(2\hat{b}\hat{\rho}\hat{b}^{+} - \hat{b}^{+}\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^{+}\hat{b}\right) + \frac{1}{2}\left(B\rho_{ac}^{(0)}\right)\left(2\hat{a}\hat{\rho}\hat{b}^{+} - \hat{\rho}\hat{b}^{+}\hat{a} - \hat{b}^{+}\hat{a}\hat{\rho}\right) + \frac{1}{2}\left(B\rho_{ac}^{(0)}\right)\left(2\hat{b}\hat{\rho}\hat{a}^{+} - \hat{a}^{+}\hat{b}\hat{\rho} - \hat{\rho}\,\hat{a}^{+}\hat{b}\right) \qquad (22)$$

*3.1.2 C-Number Langavin Equations*

The c- number Langavin equation associated to the normal orders.

$$\frac{d}{dt} < \alpha > = -\frac{1}{2}\mu_a < \alpha > -\frac{1}{2} v < \beta > \quad (23)$$

$$\frac{d}{dt} < \beta > = -\frac{1}{2}\mu_c < \beta > -\frac{1}{2} v < \alpha > \quad (24)$$

$$\frac{d}{dt} < \alpha^2 > = -\mu_a < \alpha^2 > - v < \alpha\beta > \quad (25)$$

$$\frac{d}{dt} < \beta^2 > = -\mu_c < \beta^2 > + v < \alpha\beta > \quad (26)$$

$$\frac{d}{dt} < \alpha^*\alpha > = -\mu_a < \alpha^*\alpha > -\frac{1}{2} v(< \alpha^*\beta > + < \beta^*\alpha >) \quad (27)$$

$$\frac{d}{dt} < \beta^*\beta > = -\mu_c < \beta^*\beta > -\frac{1}{2} (\mu < \alpha^*\beta > + v\beta^*\alpha >) \quad (28)$$

$$\frac{d}{dt} < \alpha^*\beta > = -\frac{1}{2}(\mu_a + \mu_c) < \alpha^*\beta > -\frac{1}{2} v(< \alpha^*\beta > + < \beta^*\alpha >) \quad (29)$$

$$\frac{d}{dt} < \alpha\beta > = -\frac{1}{2}(\mu_a + \mu_c) < \alpha\beta > -\frac{1}{2} v(< \beta^2 > + < \alpha^2 >) \quad (30)$$

On the base of this equation Eqn (23) and (24)]

$$\frac{d}{dt}\alpha(t) = -\frac{1}{2}\mu_a\alpha(t) - \frac{1}{2} v\beta(t) + f_\alpha(t) \quad (31)$$

$$\frac{d}{dt}\beta(t) = -\frac{1}{2}\mu_c\beta(t) - \frac{1}{2} v\alpha(t) + f_\beta(t) \quad (32)$$

Where $f_\alpha(t)$ and $f_\beta(t)$ are noise forces. The solution of equation (3.12) and (3.13) can be written as

$$\alpha(t) = \alpha(0)e^{\frac{-\mu_a t}{2}} - \int_0^t dt' e^{\frac{-\mu_a(t-t')}{2}} \left(\frac{1}{2} v\,\beta(t') - f_\beta(t')\right) \quad (33)$$

$$\beta(t) = \beta(0)e^{\frac{-\mu_c t}{2}} - \int_0^t dt' e^{\frac{-\mu_c(t-t')}{2}} \left(\frac{1}{2} v\,\alpha(t') - f_\beta(t')\right) \quad (34)$$

The solution of coupled differential equation Eq. (3.12) and Eq. (3.13) can be written in the matric form as

$$\frac{d}{dt}Y(t) = \frac{1}{2}MY(t) + F(t) \quad (35)$$

$$Y(t) = \begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix} \quad (36)$$

$$M = \begin{pmatrix}\mu_a & v\\ v & \mu_c\end{pmatrix} \quad (37)$$

$$F(t) = \begin{pmatrix}f_\alpha(t)\\ f_\beta(t)\end{pmatrix} \quad (38)$$

We next proceed to find the eigenvalues and eigenvectors of the matrix M. Applying the eigenvalue equation

$MU_i = \lambda U_i$ , We find the characteristic equation

$$\lambda^2 - \lambda(\mu_a + \mu_c) + \mu_c\mu_a - v^2 = 0$$

We finally obtain (39)

$$\alpha(t) = p_1(t)\,\alpha(0) + q_1(t)\beta(0) + H_1(t) \quad (40)$$

$$\beta(t) = p_2(t)\,\beta(0) + q_2(t)\alpha(0) + H_2(t) \quad (41)$$

Where

$$H_1(t) = \int_0^t p_1(t-t')f_\alpha(t') + q_1(t-t')f_\beta(t')\,dt' \quad (42)$$

$$H_2(t) = \int_0^t p_2(t-t')f_\beta(t') + q_2(t-t')f_\alpha(t')\,dt' \quad (43)$$

*3.1.3 The Q –Function*

We now proceed to determine the Q function for the cavity mode produced by a V three level lasers coupled to vacuum reservoir. The Q function for a two-mode light is expressible as

$$Q(\alpha,\beta,t) = \frac{1}{\pi^2}\int\frac{d^2z}{\pi}\frac{d^2w}{\pi}\quad \Phi(z,w,t)e^{z*\alpha - z\alpha* + w*\beta - w\beta*} \quad (44)$$

where

$$\Phi\,(z, w, t) = \mathrm{T\,r}(\hat{\rho}e^{-z*\hat{a}(t)}e^{z\hat{a}^+(t)}e^{-w*\hat{b}(t)}e^{w\hat{b}^+(t)}$$

We can write in terms of c-number variables associated to the normal ordering as

$$\Phi\,(z, w, t) = e^{-z*z}e^{-w*w} < e^{z\alpha*(t) - z*\alpha(t) + w\beta*(t) - w*\beta(t)} > \quad (45)$$

We note that α (t) and β (t) are Gaussian variables with a vanishing mean.

In the view of this finally Q-function for lambda three-level lasers,

$$Q(\alpha,\beta) = \frac{1}{\pi^2}\exp(-\alpha*\alpha - \beta*\beta) \quad (46)$$

## 3.2 Quadrature Fluctuation

The quadrature squeezing property of two-mode light is described by two Hermitian quadrature operators.

*3.2.1 Quadrature Squeezing*

Here we seek to determine the quadrature variance s for a superposed two- mode light beams produced by V- type three- level lasers. We define the quadrature variance for a superpose d two- mode light beams by

$$\Delta c^2{}_\pm = < \hat{c}_\pm, \hat{c}_\pm > \quad (47)$$

Where

$$\hat{c}_\pm = \sqrt{\pm(\hat{c}^+, \hat{c})}, \quad (48)$$

are the plus and minus quadrature operators for the superposed two-mode light beams, $\hat{c}$ and $\hat{c}^+$ are the annihilation and creation operators for the superposed two-mode light.

$$[c_+, c_-] = 8i. \quad (49)$$

In view of the relation in above equation, the superposed two-mode light is said to be in squeezed state if $either\ \Delta c - < 4\ or\ \Delta c + < 4\ such\ that\ \Delta c\ + \Delta c - \geq\ 4$. with the aid of above equation, We get

$$\Delta c^2{}_{\pm} = 4 + 2 < \hat{c}^{+}\hat{c} > \pm < \hat{c}^{+2} > \pm < \hat{c}^{2} > - 2 < \hat{c}^{+} >< \hat{c} > \mp < \hat{c}^{+} >^2 \mp < \hat{c} >^2. \quad (50)$$

Taking to account Eqn.(50) and $< \hat{c}^{+} > = < \hat{c} >$,

we have

$$\Delta c^2{}_{\pm} = 4 + 2 < \hat{c}^{+}\hat{c} > \pm 2 < \hat{c}^{2} > \quad (51)$$

Using the density operator, we write as and c- Langavin equation

$$\Delta c^2{}_{\pm} = 4 + A(1-\eta)\left[\frac{2\kappa(4\kappa+2A\eta+A)-A^2(1-\eta)}{2\kappa(2\kappa=A\eta)(\kappa+A\eta)}\right] + A\sqrt{1-\eta^2}\left[\frac{2\kappa(4\kappa+3A\eta+A)-A^2(1-\eta^2)}{2\kappa(2\kappa=A\eta)(\kappa+A\eta)}\right] \quad (52)$$

The equation represents the quadrature variation of the maximum position of the light beams produced by the Vee type of three levels. Figure 4.1 represents the variation of minus quadrature [Eq. (4.6)] compared with η in different A values. This figure shows that the compression rate increases with the coefficient of line gain and the almost total coefficient can be obtained by the maximum coefficient values of the gain line and the minimum values of η. In addition, the minimum quadrature variance defined by Eqn. (4.6) in A = 1, κ = 0.8, found 〖Δc〗 _2 ^ _ = 2.79 and occurs at η = 0.35 this result suggests that the maximum intracavity coefficient of the above values is 30.25 percent below the correlation-level of the state. [17]

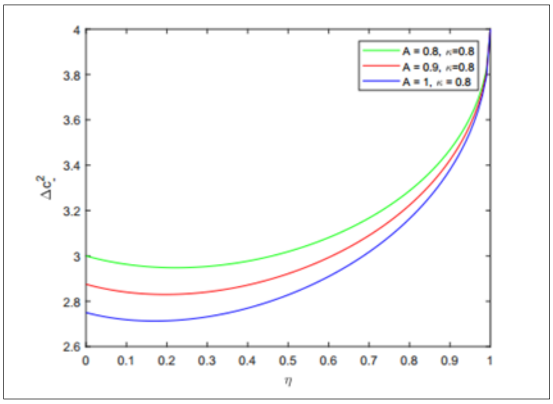


**Figure 2** Plots of the minus quadrature variance above equation versus η for κ = 0 .8, and for different values of the line a gain c o efficient.

## 3.3 Photon Statistics

In this chapter we wish to study the mathematical features of the two-dimensional superposition light lines produced by three V-level laser-operated operators that use the Q-functions of each line and the density pulse-mounted operator. [8 - 9] .

### *3.3.1 Density Operator*

Suppose ˆρ (t) be the density operator for the superposition of two beams. We can write ρˆ (t) in the normal order as

$$\hat{\rho}(t) = \sum_{ijkl} c_{ijkl}(t)\, \hat{a}^{+i}\hat{a}^{j}\hat{b}^{+k}\hat{a}^{l} \quad (53)$$

$\hat{I} = \int \frac{d^2\alpha_1}{\pi}\frac{d^2\beta_1}{\pi}|\alpha_1\beta_1\rangle\langle\beta_1\alpha_1|$, and the relation

$$|\alpha_1\beta_1\rangle\langle\beta_1\alpha_1|\, \hat{a}^{+i}\hat{b}^{+k} = a_1^{*i}b_1^{*k}|\alpha_1\beta_1\rangle\langle\beta_1\alpha_1|, \quad (54)$$

$$|\alpha_1\beta_1\rangle\langle\beta_1\alpha_1|\, \hat{a}^{+i}\hat{b}^{+k} = (\alpha_1 + \frac{\partial}{\partial\alpha_1^*})^j(\beta_1 + \frac{\partial}{\partial\beta_1^*})^k|\alpha_1\beta_1\rangle\langle\beta_1\alpha_1|$$

$\hat{\rho} = \int d\alpha_1 d\beta_1 Q_1\left(\alpha_1^*, \beta_1^*, \alpha_1 + \frac{\partial}{\partial\alpha_1^*}; \beta_1 + \frac{\partial}{\partial\beta_1^*}\right)\hat{D}(\alpha_1, \beta_1)\hat{\rho}_o \hat{D}^+(-\alpha_1 - \beta_1)$ (55)

Where

$Q_1 = \left(\alpha_1^*, \beta_1^*, \alpha_1 + \frac{\partial}{\partial\alpha_1^*}; \beta_1 + \frac{\partial}{\partial\beta_1^*}\right) = \frac{1}{2\pi}\sum_{ijkl} c_{ijkl}(t)\, \alpha_1^{*i}\beta_1^{*l}(\alpha_1 + \frac{\partial}{\partial\alpha_1^*})^j(\beta_1 + \frac{\partial}{\partial\beta_1^*})^k$ (56)

$$\hat{D}(\alpha_1, \beta_1) = e^{\alpha_1\hat{a}^+ + \beta_1\hat{b}^+ - \alpha_1^*\hat{a} - \beta_1^*\hat{b}},$$

And $\hat{\rho}_o$ is the density operator for the system at initial time

Assuming that the density operator at initial time is to be in some other state and following the same procedure to obtain the expression in Eq. (5.5), we readily find that

$\hat{\rho}_o = \int d^2\alpha_2 d^2\beta_2 Q_2\left(\alpha_2^*, \beta_2^*, \alpha_2 + \frac{\partial}{\partial\alpha_2^*}; \beta_2 + \frac{\partial}{\partial\beta_2^*}\right)\hat{D}(\alpha_2, \beta_2)|0,0\rangle\langle 0,0|\hat{D}^+(\alpha_2, \beta_2)$ (57)

In which

$Q_2 = \left(\alpha_2^*, \beta_2^*, \alpha_2 + \frac{\partial}{\partial\alpha_2^*}; \beta_2 + \frac{\partial}{\partial\beta_2^*}\right) = \frac{1}{2\pi}\sum_{pqrs} c_{pqrs}(t)\, \alpha_2^{*p}\beta_2^{*r}(\alpha_2 + \frac{\partial}{\partial\alpha_2^*})^q(\beta_2 + \frac{\partial}{\partial\beta_2^*})^s$(58)

$$\hat{D}(\alpha_2, \beta_2) = e^{\alpha_2\hat{a}^+ + \beta_2\hat{b}^+ - \alpha_1^*\hat{a} - \beta_2^*\hat{b}},$$

With the aid of Eqn. (5.5) and (5.8) as well as the relation

$\hat{D}(\alpha_2, \beta_2)\hat{D}(\alpha_1, \beta_1)|0,0\rangle\langle 0,0|\hat{D}^+(\alpha_1, \beta_1)\hat{D}^+(\alpha_2, \beta_2) = |\alpha_1 + \alpha_2 + \beta_1 + \beta_2\rangle\langle\beta_1 + \beta_2 + \alpha_1 + \alpha_2|$
(59)

The expectation value of the annihilation operator for two-mode superposed light beams represented by ˆc in terms of density operator is given by

$\langle\hat{a}\rangle = Tr(\hat{\rho}(t)\hat{c}(0))$ (60)

Taking account Eqn. () and $\int d^2\alpha_i d^2\beta_i Q_i\left(\alpha_i^*, \beta_i^*, \alpha_i + \frac{\partial}{\partial\alpha_i^*}; \beta_i + \frac{\partial}{\partial\beta_i^*}\right) = 1$, we have

$\langle\hat{c}\rangle = \langle\hat{a}_1\rangle + \langle\hat{b}_1\rangle + \langle\hat{a}_2\rangle + \langle\hat{b}_2\rangle$ (61)

$\langle\hat{a}_i\rangle = \int d^2\alpha_i d^2\beta_i Q_i\left(\alpha_i^*, \beta_i^*, \alpha_i + \frac{\partial}{\partial\alpha_i^*}; \beta_i + \frac{\partial}{\partial\beta_i^*}\right)\alpha_i$ (62)

$\langle\hat{b}_i\rangle = \int d^2\alpha_i d^2\beta_i Q_i\left(\alpha_i^*, \beta_i^*, \alpha_i + \frac{\partial}{\partial\alpha_i^*}; \beta_i + \frac{\partial}{\partial\beta_i^*}\right)\beta_i$ (63)

*with* $i = 1,2$. Based on the result given by Eqn. (63), one can write the operator representing the superposed light beams as [9]

$\hat{c} = \hat{a}_1 + \hat{a}_2 + \hat{b}_1 + \hat{b}_2$ (64)

With the commutation relation
$[\hat{c}, \hat{c}^+] = 4.$ (65)

We note that the Q-function for a single mode can obtained using the relation

$Q_i(\alpha_i^*, \alpha_i) == \int d^2\beta_i Q_i(\alpha_i, \beta_i, t)$ (66)

$$Q_i(\alpha_i^*, \alpha_i + \frac{\partial}{\partial\alpha_i^*}) = Q_i(\alpha_i^*, \alpha_i, t)e^{-\mathrm{U}a_i\alpha_i^*\frac{\partial}{\alpha_i^*}}, \quad (67)$$

$$Q_i(\beta_i^*, + \frac{\partial}{\partial\beta_i^*})\beta_i = Q_i(\beta_i^*, \beta_i, t)e^{-\mathrm{U}a_i\beta_i^*\frac{\partial}{\beta_i^*}} \quad (68)$$

*3.3.2 Mean Photon Number*

In this section we want to calculate the average number of photon f or two-dimensional light rays produced by cascades and three V-level lasers. The description of the photon number for the superposition of the two light lines can be displayed depending on the density operator as [13]

$$\bar{n} = \langle\hat{c}^+\hat{c}\rangle = Tr(\hat{\rho}\hat{c}^+\hat{c}) \quad (69)$$

$$\hat{C}^+(t)\hat{c}(t) = \int d^2\alpha_1 d^2\alpha_2 d^2\beta_1 d^2\beta_2 \mathrm{Q}_1\left(\alpha_1^*, \beta_1^*, \alpha_1 + \frac{\partial}{\partial\alpha_1^*}; \beta_1 + \frac{\partial}{\partial\beta_1^*}\right) \left(\alpha_2^*, \beta_2^*, \alpha_2 + \frac{\partial}{\partial\alpha_2^*}; \beta_2 + \frac{\partial}{\partial\beta_2^*}\right)|\alpha_1 + \alpha_2 + \beta_1 + \beta_2|^2 \quad (70)$$

This equation can be put in the form

$$\hat{C}^+(t)\hat{c}(t) = \langle a_1^+\hat{a}_1\rangle + \langle a_2^+\hat{a}_2\rangle + \langle\hat{b}_1^+\hat{b}_1\rangle + \langle\hat{b}_2^+\hat{b}_2\rangle + \langle a_1^+\hat{b}_1\rangle + \langle\hat{b}_1^+\hat{a}_1\rangle$$

$$+\langle\hat{b}_2^+\hat{a}_2\rangle + \langle\hat{a}_2^+\hat{b}_2\rangle + \langle\hat{b}_2^+\hat{a}_2\rangle + \langle\hat{a}_1^+\rangle\langle\hat{a}_2\rangle + \langle\hat{a}_2^+\rangle\langle\hat{a}_1\rangle + \langle\hat{a}_1^+\rangle\langle\hat{b}_2\rangle$$

$$+\langle\hat{b}_2^+\rangle\langle\hat{a}_1\rangle + \langle\hat{b}_1^+\rangle\langle\hat{a}_2\rangle + \langle\hat{a}_2^+\rangle\langle\hat{b}_1\rangle + \langle\hat{b}_1^+\rangle\langle\hat{b}_2\rangle + \langle\hat{b}_2^+\rangle\langle\hat{b}_1\rangle \quad (71)$$

Carrying out the integration and then performing the differentiation, we find

$$\langle\hat{a}_i^+\hat{a}_i\rangle = a_1 - 1 \quad (72)$$

$$\langle a_1^+\hat{a}_1\rangle = a_1 - 1 \quad \langle a_2^+\hat{a}_2\rangle = 0 \quad (73)$$

Similarly

$$\langle\hat{b}_i^+\hat{b}_i\rangle = b_1 - 1 \quad (74)$$

$$\langle\hat{b}_1^+\hat{b}_1\rangle = b_1 - 1, \langle\hat{b}_2^+\hat{b}_2\rangle = o \quad (75)$$

$$\langle\hat{a}_i\rangle = \langle\hat{b}_i\rangle = \langle\hat{a}_i^+\hat{b}_i\rangle = 0 \quad (76)$$

Then finally

$$\hat{C}^+(t)\hat{c}(t) = \sum_1^2\langle\hat{a}_i^+\hat{a}_i\rangle + \langle\hat{b}_i^+\hat{b}_i\rangle \quad (77)$$

Eq. (77) represents the medium-sized photon number at the top of the cascade with three lambda level levels. We see in Eq. (77) that the direct photon number of superposition light lines is the sum of a three-level cascade photon number and a V-type three-level laser photon number. [6]

## 4 Conclusion

In this study, we coupled a thermal reservoir with a three-level laser to monitor the statistics and congestion of the light output across all its channels. Atoms are injected into a hole connected to a cleaning dam through a single mirror hole, initially placed at high levels and arranged in a cascade with three levels. We derived the primary equation for the three-level laser light using the line and adiabatic balancing system. From there, we obtained the Langevin c-number equations and their solutions. These methods allowed us to determine a specific feature function for ants, which we then used in Vee to calculate the Q function of the light emitted by a three-level laser. Using the Q function, we computed the quadrature variation and average image number. Furthermore, we calculated the three-dimensional Vee superposition-type laser Q function. By utilizing this function, we determined the quadrature variation, average image number, and density operator to achieve a stable position. It has been observed that as the line gain coefficient increases, the compression rate also increases. With small values of η, it is possible to achieve nearly absolute pressure. Our ultimate finding suggests that while Vee quadrature variations remain consistent, there may be additional factors at play. congestion in a cascade compared to an extreme scenario.

In summary, the properties of the two-mode cavity radiation can be manipulated by controlling various factors such as the injected atomic coherence, linear gain coefficient, amplitude of the driving coherent light, and the presence of thermal light. Squeezing, which refers to reducing noise in one quadrature of the radiation, can be increased by raising the linear gain coefficient and the amplitude of the driving coherent light. However, the linear correlation coefficient, which characterizes the correlation between the two quadratures, increases with the degree of squeezing. On the other hand, the presence of thermal light has a detrimental effect on squeezing, causing it to decrease. This implies that the level of noise in the radiation increases with the amount of thermal light present.

Furthermore, the parametric amplifier, a device that amplifies signals based on a nonlinear interaction, is capable of generating a significant amount of entangled light. This occurs even for very small values of the linear gain coefficient, regardless of the initial preparation of the atoms. The level of entanglement in the two-mode cavity light is directly linked to the degree of two-mode squeezing. Overall, these findings emphasize the intricate relationship between various factors and the properties of the two-mode cavity radiation, including squeezing, correlation, and entanglement. Quantum discords are a measure of quantum correlations that differ from entanglement. They quantify the extent of non-classical correlations between two subsystems of a quantum system. Investigating the quantum discords of the two-mode cavity radiation and how they are influenced by factors such as squeezing and thermal light would be interesting. This could provide further understanding of the nature of quantum correlations in the system and how they can be manipulated. Overall, studying the properties of the two-mode cavity radiation has significant implications for quantum information processing and communication, and further research in this area is justified.

## Compliance with ethical standards

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*Disclosure of conflict of interest*

The authors declare that there are no conflicts of interest regarding the publication of this paper.

*Data Availability*

The data used to support the findings of this study are included within the manuscript.

## References

[1] Fesseha Kassahun, Basic Quantum Optics (Lulu Press Inc., North Carolina, 2008).

[2] Marlan O.Scully and M.Suhail Zubairy, Quantum Optics (University of Cambridge, 1997).

[3] Mark Fox, Introduction to Quantum Optic, (Oxford University press, 2006).

[4] Misrak Getahun, Third Level Laser Dynamic Integrated Light, PhD Dissertation (Addis Ababa University, 2009).

[5] Fesseha Kassahun, Three-Level Laser Dynamics Atom-Electrized Atoms Bombardment, (Addis Ababa University, 2012).

[6] Assegid Mengistu, MSc Thesis, (Addis Ababa University, 2010).

[7] S. M. Barnett and P. M. Raymore, Methods in Theoretical Quantum Optic s, (Oxford,University Press, New York, 1997)

[8] TewodrosYirgeshewa Darge, Shared Third Level Lase Parametric Magnificent, PhD Dissertation (Addis Ababa University, 2010)

[9] M. O. Scully and M. S. Zubairy, Quantum Optics, (Cambridge University Press, 1997).

[10] The Wobshet Mekonen is a three-level non-invasive laser with cavity-driven modeCoherence light MSc Thesis, (Addis Ababa University, 2007).

[11] Sintayehu Tesfa, arxiv: 07082815v1 [quanta-ph. (2007).

[12] Sintayehu Tesfa a No degenerate Three-level Cascade Laser Couple d to a Two- mode Squeezed Vacuum Reservoir PhD Dissertation (Addis Ababa University 2008)

[13] Economou, Sophia Automatic Extrusion and Visual Control of Spins in Quantum Dots PhD Dissertation (UNIVE RSIT Y OF CALIFORNI A, 2006)

[14] Beyene Abiti Superposed De gene rate Three-Level Lasers MSc thesis (Addis Ababa University 2011)

[15] Tizazu Maresha Superposition of Squeezed Laser Light Beams PhD Dissertation (Addis Ababa University 2015)

[16] Solomon Getachew TWO MODE COHERENT MODES AND LASER LIG HT BE AMS MScThesis (Addis Ababa University June 2011)

[17] Peter Lambropoulos David Petrosyan Basic Quantum Optics and Quantum (Springer) Knowledge